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# STUDY OF THERMAL BUOYANCY AND CONCENTRATION BUOYANCY EFFECTS ON THE ELECTRICALLY CONDUCTING FLUID FLOW PAST AN IMPULSIVELY STARTED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION IN THE PRESENCE OF INCLINED MAGNETIC FIELD.

**DEVA KANTA PHUKAN**
Former Principal cum Professor,
Demow college(SVR)
Dibrugarh, Assam, India-786001

**Abstract**

Thermal buoyancy and concentration buoyancy effects of the conducting fluid past an impulsively started vertical plate with variable temperature and mass diffusion in the presence of inclined magnetic field is studied in this investigation. Assuming the fluid is gray, absorbing heat and emitting radiation in a non-scattering medium. The dimensional governing partial differential equations obtained in this investigation are converted to non- dimensional partial differential equations and are solved by using Laplace-transform technique subject to the respective boundary conditions. The graphical results are depicted for velocity, temperature and concentration profiles for various existing parameters and time in this discursion. The skin friction for different magnetic parameter, Prandtl number, Schmidt number, thermal Grashof number and mass Grashof number.

Keywords : *Unsteady flow, Vertical plate, Conducting fluid, Magnetic field, Heat transfer, Mass transfer.*

2000 AMS Subject Classification : 76W05, 80A20, 76 N20, 44A10.

## 1. Introduction

During recent years, simultaneous study of conducting fluid flow with heat and mass transfer in presence of magnetic field in different geometries and in various modelling have been observed to play an important role in many engineering and geophysical applications, chemical reaction and biological Science. Some important applications are cooling of nuclear reactors, Liquid metals fluid, power generation system and aerodynamics. The response of laminar skin friction and heat transfer to fluctuations in the stream velocity was first studied by Lighthill [1].]. The free convection effects on the oscillatory flow past an infinite vertical porous plate with constant suction was presented by Soundelgekar [2] . Gupta et al.[3] have studied free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid. Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux was investigated by Jha et al.[4]. An investigation on radiation effects on free convection flow past a semi-infinite vertical plate was presented by Soundelgekar and Takhar [5]. Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction was studied by Das et al [6]. Several researchers in these areas on heat transfer and radiation, mass transfer and chemical reaction were investigated in different models. One may refer the works presented by Soundelgekar and Takhar [6] ,Hossain and Takhar [8], Rapit and Perdikis [9] and Muthucumaraswamy et. al [12]. MHD and Radiation effects on moving isothermal vertical plate with variable mass diffusion have been studied by Muthucumaraswamy and Janakiraman [10]. Prasad et al. [11] investigated radiation and mass transfer effects on two V dimensional flow past an impulsively started infinite vertical plate. Furthermore, these types of works in areas of MHD effects on free convection, radiation effects on mixed convection, combined effects of radiation and mass transfer have been studied. It may be inferred the works done by Muthucumaraswamy[12], Rajesh [13] and Rajput and Surendra [14].

In the present investigation, we have considered to discuss the thermal buoyancy and concentration buoyancy effects on the electrically conducting fluid flow past an impulsively started vertical plate with variable temperature and mass transfer in the presence of inclined magnetic field. The governing partial differential equations have been converted to non-dimensional partial differential equations for momentum, energy and concentration and are solved by using Laplace transform and inverse Laplace transform technique for different parameters existing in this investigation.

## 2. Formulation of the Problem

We consider the flow of unsteady, viscous, incompressible electrically conducting fluid past an impulsively started vertical plate. The $x$-axis is taken along the plate and $y$-axis is taken perpendicular to the plate. The temperature of the plate and fluid are at the same initially i.e. before starting the motion. An inclined magnetic is applied through the point of intersection of $x$-axis and y-axis making an angle $\alpha$ in the positive direction of $x$-axis. The viscous dissipation and induced magnetic field has been ignored due to its less effect. Initially the fluid and plate are at the same temperature $T_\infty$ and concentration $C_\infty$ in the stationary condition. At $t > 0$, the temperature of the plate is raised to $T_w$ and concentration level near the plate is raised to $C_w$ linearly with respect to time. Since the plate is considered in the $x$-direction, and hence all physical quantities will be independent of $x$. So under these assumption, the physical variables are purely the function of $y$ and $t$ only.

The mathematical model of the flow is as below

$$\frac{\partial u}{\partial t} = g\beta_t(T - T_\infty) + g\beta_m(C - C_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 Cos^2\alpha}{\rho}u, \tag{1}$$

$$\frac{\partial T}{\partial t} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2}, \tag{2}$$

and

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

where $u$ is the velocity of the fluid in $x$ - direction, $y$ - co-ordinate axis norm al to the plate, $g$ - acceleration due to gravity, $t$ -time , $\rho$-density of the fluid, $\beta_t$ - volumetric co-efficient of thermal expansion, $\beta_m$-volumetric co-efficient of concentration expansion, $\sigma$ -electrical conductivity of the fluid(Stefan Boltzmann constant), $\nu$ -kinematics viscosity, $T_\infty$ -temperature of the fluid near the plate, $C$ -species concentration in the fluid, $T$ -temperature of the fluid far from the plate, $C_\infty$ -concentration in the fluid far away from the plate, $B_0$ -external magnetic field, $\kappa$ -thermal conductivity of the fluid, $C_p$ specific heat at constant pressure and $D$ -mass diffusion.

The boundary conditions for the above governing equations are as follows

$$\left.\begin{aligned} & t \leq 0 : u = 0, T = T_\infty, C = C_\infty, \quad \text{for all values of } y \quad i.e. \ \forall y \\ & t > 0 : u = u_0, T = T_\infty + (T_w - T_\infty)\tfrac{u_0^2 t}{\nu}, C = C_\infty + (C_w - C_\infty)\tfrac{u_0^2 t}{\nu}, \ at \ y = 0, \\ & \qquad u \to u_0, T \to T_\infty, \quad C \to C_\infty \quad as \quad y \to \infty. \end{aligned}\right\} \tag{4}$$

Introducing the following non-dimensional quantities

$$\left.\begin{aligned} & \overline{y} = \tfrac{y u_0}{\nu}, \overline{u} = \tfrac{u}{u_0}, \theta = \tfrac{T - T_\infty}{T_w - T_\infty}, \phi = \tfrac{C - C_\infty}{C_w - C_\infty}, \overline{t} = \tfrac{t u_0^2}{\nu}, \\ & M = \tfrac{\sigma B_0^2 \nu}{\rho u_0^2}, P_r = \tfrac{\mu C_p}{\kappa}, S_c = \tfrac{\nu}{D}, G_r = \tfrac{g \beta_t \nu (T_w - T_\infty)}{u_0^3}, G_r = \tfrac{g \beta_m \nu (C_w - C_\infty)}{u_0^3}. \end{aligned}\right\} \tag{5}$$

Using equation (5) in equations(1)-(3) we get the respective following non- dimensional equation of momentum, energy and concentration of the conducting fluid as

$$\frac{\partial \overline{u}}{\partial \overline{t}} = \frac{\partial^2 \overline{u}}{\partial \overline{y^2}} + G_r \theta + G_m \phi - (M \cos^2 \alpha) \overline{u}, \tag{6}$$

$$\frac{\partial \overline{\theta}}{\partial \overline{t}} = \frac{1}{P_r} \frac{\partial^2 \overline{\theta}}{\partial \overline{y^2}} \tag{7}$$

and

$$\frac{\partial \overline{\phi}}{\partial \overline{t}} = \frac{1}{S_c} \frac{\partial^2 \overline{\phi}}{\partial \overline{y^2}}. \tag{8}$$

Subject to the following boundary conditions

$$\left.\begin{aligned} & \overline{t} \leq 0 : \overline{u} = 0, \theta = 0, \phi = 0, \quad \text{for all vaules of } \ \overline{y} \quad i.e. \ \forall \ \overline{y} \\ & \overline{t} > 0 : \overline{u} = 1, \theta = \overline{t}, \phi = \overline{t} \quad at \quad \overline{y} = 0, \\ & \qquad \overline{u} \to 0, \ \theta \to 0, \phi \to 0 \quad as \quad \overline{y} \to \infty. \end{aligned}\right\} \tag{9}$$

If we drop bars in the afore said non dimensional equations, we have

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_m \phi - (M \cos^2 \alpha) u, \tag{10}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} \tag{11}$$

and

$$\frac{\partial \phi}{\partial t} = \frac{1}{S_c} \frac{\partial^2 \phi}{\partial y^2}. \tag{12}$$

with the following appropriate boundary conditions

$$\left.\begin{aligned} &t \le 0 : u = 0, \theta = 0, \phi = 0, \quad \text{for all values of} \;\; y \;\; i.e. \;\; \forall y \\ &t > 0 : u = 1, \theta = t, \phi = t \;\; at \;\; y = 0, \\ &\quad u \to 0, \theta \to 0, \phi \to 0 \;\; as \quad y \to \infty. \end{aligned}\right\} \tag{13}$$

The non-dimensional governing equations (10)-(12) subject to the boundary conditions (13) have been solved by using usual Laplace transform technique along with the help of B. Hetnarskis [15] development made on Laplace transform and inverse Laplace transform technique.

First of all, using Laplace transform technique in concentration equation (12), we have

$$L\left\{\frac{\partial \phi(y,t)}{\partial t}\right\} = L\left\{\frac{1}{S_c}\frac{\partial^2 \phi(y,t)}{\partial y^2}\right\},$$

which implies

$$sL\{\phi(y,t)\} - \phi(y,0) = \frac{1}{S_c}L\left\{\frac{\partial^2 \phi(y,t)}{\partial y^2}\right\}. \tag{14}$$

Using the boundary conditions (13) in (14) we get,

$$\frac{d^2 L\{\phi(y,t)\}}{dy^2} - sS_c L\{\phi(y,t)\} = 0,$$

which gives the solution as below

$$L\{\phi(y,t)\} = A_1 e^{y\sqrt{sS_c}} + A_2 e^{-y\sqrt{sS_c}}, \tag{15}$$

where $A_1$ and $A_2$ are arbitrary constants.

Again using the boundary conditions (13), we get

$$L\{\phi(y,t)\} = L\{t\} = \frac{1}{s^2} = A_1 + A_2. \tag{16}$$

Since $\phi(y,t)$ is bounded for $y \to \infty$, so $\phi(y,s)$ is also bounded as $y \to \infty$ which implies that we must choose $A_1$ as Spiegel, M. R. Spiegel[16] Page 97

$$L\{\phi(y,t)\} = L\{0\} = 0 = A_1. \tag{17}$$

Using equations (16) and (17), in equation (15) gives the following solution

$$L\{\phi(y,t)\} = \frac{1}{s^2}e^{-y\sqrt{sS_c}}. \tag{18}$$

Using inverse Laplace transform technique, we get,

$$\phi(y,t) = L^{-1}\left\{\frac{1}{s^2}e^{-y\sqrt{sS_c}}\right\}.,$$

$$\Rightarrow \phi(y,t) = \left(t + \frac{y^2 s_c}{2} erfc\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}\right)\right) - \frac{y\sqrt{S_c t}}{\sqrt{\pi}} e^{-\frac{y^2 S_c}{4t}}. \tag{19}$$

Again transforming the equation (11) and using the boundary (13), we get

$$\frac{d^2 L\{\theta(y,t)\}}{dy^2} - sP_r L\{\theta(y,t)\} = 0,$$

whose solution gives after using boundary conditions(13) as follows.

$$L\{\theta(y,t)\} = A_3 e^{y\sqrt{sP_r}} + A_4 e^{-y\sqrt{sP_r}},$$

$A_3$ and $A_4$ are arbitrary constants.
Using (13), values of $A_3$ and $A_4$ may be calculated as $A_3 = 0$ and $A_4 = \frac{1}{s^2}$ which implies

$$L\{\theta(y,t)\} = \frac{1}{s^2} e^{-y\sqrt{sP_r}}. \tag{20}$$

Now taking inverse Laplace transform, we get

$$\theta(y,t) = \left(t + \frac{y^2 P_r}{2} erfc\left(\frac{y\sqrt{P_r}}{2\sqrt{t}}\right)\right) - \frac{y\sqrt{P_r t}}{\sqrt{\pi}} e^{-\frac{y^2 P_r}{4t}}. \tag{21}$$

Again taking Laplace transform on both sides of equation (10), we have

$$L\left\{\frac{\partial u(y,t)}{\partial t}\right\} = L\left\{\frac{\partial^2 u(y,t)}{\partial y^2} + G_r\theta(y,t) + G_m\phi(y,t) - (M\cos^2\alpha)u(y,t)\right\}.$$

which implies $sL\{u(y,t)\} - u(y,0)$

$$= \frac{d^2 L\{u(y,t)\}}{dy^2} - (M\cos^2\alpha)L\{u(y,t)\} + G_r L\{\theta(y,t)\} + G_m L\{\phi(y,t)\}. \tag{22}$$

Using the boundary conditions (13), equation (22) gives

$$\frac{d^2 L\{u(y,t)\}}{dy^2} - (s + M\cos^2\alpha)L\{u(y,t)\} = -G_r L\{\theta(y,t)\} - G_m L\{\phi(y,t)\}. \tag{23}$$

The general solution of (23) will be of the form

$$L\{u(y,t)\} = A.E. + P.I. \tag{24}$$

with

$$A.E. = A_5 e^{y\sqrt{s+M\cos^2\alpha}} + A_6 e^{-y\sqrt{s+M\cos^2\alpha}},$$

and $A_5$ and $A_6$ are arbitrary constants and

$$P.I. = -G_r \frac{1}{D^2 - s + M_1} L\{\theta(y,t)\} - G_m \frac{1}{D^2 - s + M_1} L\{\phi(y,t)\}$$

where $M_1 = M\cos^2\alpha$. Substituting the value of $L\{\phi(y,t)\}$ and $L\{\theta(y,t)\}$ from equation (18) and (20) respectively, equation (24) becomes

$$L\{u(y,t)\} = A_5 e^{y\sqrt{s+M\cos^2\alpha}} + A_6 e^{-y\sqrt{s+M\cos^2\alpha}} + \frac{G_r}{(1-P_r)} \frac{e^{-y\sqrt{sP_r}}}{s^2(s-a)} + \frac{G_m}{(1-S_c)} \frac{e^{-y\sqrt{sS_c}}}{s^2(s-b)}, \tag{25}$$

where $a = \frac{M_1}{P_r - 1}$ and $b = \frac{M_1}{S_c - 1}$.

Taking inverse Laplace transform of equation (25) and using boundary conditions (13) yields

$$\frac{1}{a} = A_5 + A_6 + \frac{G_r}{(1-P_r)s^2(s-a)} + \frac{G_m}{(1-S_c)s^2(s-b)}.$$

When $y \to \infty$, $u(y,t) \to 0$, so $L\{u(y,t)\} \to 0$ for $y \to \infty$ and hence $L\{u(y,t)\}$ is bounded for being $u(y,t)$ is bounded when $y \to \infty$. So $A_5 = 0$ and

$$A_6 = \frac{1}{s} - \frac{G_r}{(1-P_r)s^2(s-a)} - \frac{G_m}{(1-S_c)s^2(s-b)}.$$

Hence equation (25) becomes

$$\begin{aligned} L\{u(y,t)\} = {} & \frac{1}{s} e^{-y\sqrt{s+M_1}} + \\ & \frac{G_r}{(1-P_r)} \frac{(e^{-y\sqrt{sP_r}} - e^{-y\sqrt{s+M_1}})}{s^2(s-a)} \\ & + \frac{G_m}{(1-S_c)} \frac{(e^{-y\sqrt{sSc}} - e^{-y\sqrt{s+M_1}})}{s^2(s-b)}. \end{aligned} \tag{26}$$

Taking inverse Laplace transform of (26) and with the help of Article by Hetnarski, R.B. [15] the solutions are derived as below.

$$
\begin{aligned}
u(y,t) = {} & B_1 e^{-y\sqrt{M_1}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{M_1 t}\right) + B_2 e^{y\sqrt{M_1}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{M_1 t}\right) \\
& -G_3 erfc\left(\frac{y\sqrt{P_r}}{2\sqrt{t}}\right) - G_4 erfc\left(\frac{y\sqrt{S_c}}{2\sqrt{t}}\right) + G_5\left\{e^{-y\sqrt{aP_r}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{at}\right)\right. \\
& +e^{y\sqrt{aP_r}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{at}\right) + e^{-y\sqrt{a+M_1}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(a+M_1)t}\right) \\
& \left.+e^{y\sqrt{a+M_1}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(a+M_1)t}\right)\right\} \\
& +G_6\left\{e^{-y\sqrt{bS_c}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{bt}\right) + e^{y\sqrt{bS_c}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{bt}\right)\right\} \\
& -G_6\left\{e^{-y\sqrt{b+M_1}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(b+M_1)t}\right) + e^{y\sqrt{b+M_1}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(b+M_1)t}\right)\right\} \\
& -G_7\left\{t + \frac{y^2 P_r}{2} erfc\left(\frac{yP_r}{2\sqrt{t}}\right) - \frac{y\sqrt{P_r t}}{\sqrt{\pi}} e^{-\frac{y^2 P_r}{4t}}\right\} \\
& -G_8\left\{t + \frac{y^2 S_c}{2} erfc\left(\frac{yS_c}{2\sqrt{t}}\right) - \frac{y\sqrt{S_c t}}{\sqrt{\pi}} e^{-\frac{y^2 S_c}{4T}}\right\} \qquad (27)
\end{aligned}
$$

where

$$
G_1 = 1 + G_3 + G_4, \quad G_2 = G_7 + G_8, \quad G_3 = \frac{G_r}{a^2(1-P_r)}, \quad G_4 = \frac{G_m}{b^2(1-S_c)},
$$

$$
G_5 = \frac{G_3}{2} e^{at}, \quad G_6 = \frac{G_4}{2} e^{bt}, \quad G_7 = \frac{G_r}{a(1-P_r)}, \quad G_8 = \frac{G_m}{b(1-S_c)},
$$

$$
B_1 = \frac{1}{2}\left[G_1 + G_2\left(t - \frac{y}{2\sqrt{M_1}}\right)\right], \quad B_2 = \frac{1}{2}\left[G_1 + G_2\left(t + \frac{y}{2\sqrt{M_1}}\right)\right].
$$

**Skin friction, Nusselt number and Sherwood number** : In this paper, we consider the physical quantities of interaction which are known as the Skin friction co-efficient $C_f$, the Nusselt number $N_u$ and the Shewood number $S_h$ which are defined as $N_u = \frac{\nu q_w}{u_0 \kappa (T_w - T_\infty)}$ and $S_h = \frac{\nu j_w}{u_0 D (C_w - C_\infty)}$ respectively where $\tau_w$ is the skin friction or shear stress, $q_w$ and $j_w$ are the heat and mass flux from the plate which are given by $\tau_w = -\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$, $q_w = -\alpha\left(\frac{\partial T}{\partial y}\right)_{y=0}$, and $j_w = -D\left(\frac{\partial C}{\partial y}\right)_{y=0}$, respectively.
Using the non-dimensionless variables (5) the dimensionless physical parameters skin-friction coefficient, Nusselt number and Sherwood number are found as below

Skin frictionco efficient :

$$
\begin{aligned}
\tau_w &= \frac{1}{2M^2}\sqrt{M}(erfc\sqrt{Mt}-2)G_r(P_r-1)+\frac{1}{2M^2}G_m(S_c-1)+\frac{t(G_m+G_r)}{2M} \\
&+\frac{1}{4M\sqrt{M}}(G_m+G_r)(erfc\sqrt{Mt}-2)+G_r\sqrt{\frac{P_r t}{\pi}}+\frac{t}{M}\sqrt{\frac{P_r}{\pi t}}+G_m\sqrt{\frac{S_c t}{\pi}} \\
&+\frac{t}{M}\sqrt{\frac{S_c}{\pi t}}+\frac{1}{2M^2}\sqrt{M}erfc\sqrt{Mt}G_r(P_r-1)+\frac{t(G_m+G_r)}{2M} \\
&+\frac{1}{4M\sqrt{M}}(G_m+G_r)erfc\sqrt{Mt}-\frac{1}{2M^2}G_r(P_r-1)e^{-Mt}+\frac{1}{2M^2}G_m(S_c-1) \\
&+\frac{t(G_m+G_r)}{2M\sqrt{\pi t}}-\frac{1}{2M^2}G_r(P_r-1)e^{-Mt}+\frac{t(G_m+G_r)}{2M\sqrt{\pi t}} \\
&+\frac{1}{2M^2}\left[G_r e^{-\frac{Mt}{P_r-1}}(P_r-1)erfc\left(\sqrt{t\left(M+\frac{M}{P_r-1}\right)}\right)\sqrt{\left(M+\frac{M}{P_r-1}\right)}\right. \\
&+\left\{erfc\left(\sqrt{t\left(M+\frac{M}{P_r-1}\right)}\right)-2\right\}\sqrt{\left(M+\frac{M}{P_r-1}\right)} \\
&\left.-\frac{2e^{-t\left(M+\frac{M}{P_r-1}\right)}}{\sqrt{\pi t}}\right]-\frac{1}{2M^2}\left[G_m e^{-\frac{Mt}{S_c-1}}\right. \\
&erfc\left(\sqrt{t\left(M+\frac{M}{S_c-1}\right)}\right)\sqrt{\left(M+\frac{M}{S_c-1}\right)}+\left\{\left(\sqrt{t\left(M+\frac{M}{S_c-1}\right)}\right)-2\right\} \\
&\left.\sqrt{\left(M+\frac{M}{P_r-1}\right)}-\frac{2e^{-t\left(M+\frac{M}{S_c-1}\right)}}{\sqrt{\pi t}}\right] \\
&+\frac{1}{2M^2}\left[G_r e^{-\frac{Mt}{P_r-1}}(P_r-1)\left\{erfc\left(\sqrt{\frac{Mt}{P_r-1}}\right)-2\right\}\right. \\
&\left.\sqrt{\frac{MP_r}{P_r-1}}+erfc\left(\sqrt{\frac{Mt}{P_r-1}}\right)-2\frac{e^{-\frac{Mt}{P_r-1}}}{\sqrt{\pi t}}\right] \\
&+\frac{1}{2M^2}\left[G_m e^{-\frac{Mt}{S_c-1}}(S_c-1)\left\{\left(\sqrt{\frac{Mt}{S_c-1}}\right)-2\right\}\right. \\
&\left.\sqrt{\frac{Mt}{S_c-1}}+erfc\left(\sqrt{\frac{Mt}{S_{cr}-1}}\right)\sqrt{\frac{MS_c}{S_c-1}}-2\frac{e^{-\frac{Mt}{S_c-1}}}{\sqrt{\pi t}}\right] \\
&+\frac{G_r(P_r-1)\sqrt{P_r}}{M^2\sqrt{\pi t}}+\frac{G_m(S_c-1)\sqrt{S_c}}{M^2\sqrt{\pi t}} \qquad (28)
\end{aligned}
$$

Nusselt number :

$$N_u = -\sqrt{\frac{P_r t}{\pi}} \tag{29}$$

Sherwood number :

$$S_h = -\sqrt{\frac{S_c t}{\pi}} \tag{30}$$

**Table 1** : Effects of magnetic parameter $M$ on Skin friction, Nusselt number and Sherwood number for $P_r = 1.2, S_c = 1.5, G_m = 2, t = 0.5, G_r = 1$

| $M$ | $\tau_w$ | $Nu$ | $Sh$ |
|---|---|---|---|
| 0.5 | 13.7897 | 0.6308 | 0.4370 |
| 1 | 6.4596 | 0.6308 | 0.4370 |
| 2 | 4.2994 | 0.6308 | 0.4370 |
| 3 | 4.0184 | 0.6308 | 0.4370 |

**Table 2** : Effects of Prandtl number $P_r$ on Skin friction, Nusselt number and Sherwood number for $M = 2, S_c = 1.5, G_m = 2, t = 0.5, G_r = 1$.

| $P_r$ | $\tau_w$ | $Nu$ | $Sh$ |
|---|---|---|---|
| 1.4 | 8.2211 | 0.4720 | 0.4370 |
| 2 | 4.4320 | 0.5642 | 0.4370 |
| 2.5 | 4.2994 | 0.6308 | 0.4370 |
| 7 | 5.3796 | 1.0555 | 0.4370 |

**Table 3** : Effects of Schmidt number $S_c$ on Skin friction, Nusselt number and Sherwood number for $P_r = 1.2, M = 2, G_m = 2, t = 0.5, G_r = 1$.

| $S_c$ | $\tau_w$ | $Nu$ | $Sh$ |
|---|---|---|---|
| 1.2 | 4.2994 | 0.6308 | 0.4370 |
| 2 | 4.0991 | 0.6308 | 0.5642 |
| 3 | 3.6403 | 0.6308 | 0.6910 |
| 4 | 2.9946 | 0.6308 | 0.7979 |

**Table 4** : Effects of thermal buoyancy parameter $G_r$ on Skin friction, Nusselt number and Sherwood number for $P_r = 1.2, S_c = 1.5, G_m 2, t = 0.5, M = 2$.

| | | | |
|---|---|---|---|
| 1 | 4.2994 | 0.6308 | 0.4370 |
| 2 | 6.9276 | 0.6308 | 0.4370 |
| 5 | 14.8122 | 0.6308 | 0.4370 |

**Table 5** : Effects of concentration buoyancy parameter $G_m$ on Skin friction, Nusselt number and Sherwood number for $P_r = 1.2, S_c = 1.5, M = 2, t = 0.5, G_r = 1$.

| $G_m$ | $\tau_w$ | $Nu$ | $Sh$ |
|---|---|---|---|
| 1 | 4.1887 | 0.6308 | 0.4370 |
| 2 | 4.2994 | 0.6308 | 0.4370 |
| 3 | 4.4101 | 0.6308 | 0.4370 |
| 5 | 4.6315 | 0.6308 | 0.4370 |

**Table 6** : Effects of time $t$ on Skin friction, Nusselt number and Sherwood number for $P_r = 1.2, S_c = 1.5, G_m = 2, M = 2, G_r = 1$.

| $t$ | $\tau_w$ | $Nu$ | $Sh$ |
|---|---|---|---|
| 0.1 | 3.3737 | 0.2821 | 0.1954 |
| 0.3 | 3.5633 | 0.4886 | 0.3385 |
| 0.5 | 4.2994 | 0.6308 | 0.4370 |
| 0.7 | 4.3039 | 0.7464 | 0.5171 |
| 1 | 7.3584 | 0.8921 | 0.6180 |

## 3. Results and Discussion

The values of skin friction coefficient, Nusselt number and Sherwood numbers calculated from (28) - (30) for different existing parameters are tabulated in Table 1-6. It is clearly observed from Table-1 & 3 that magnitude of skin-friction co-efficient decreases as the magnetic parameter $M$ and Schmidt number $S_c$ increase and no magnetic effects are observed on Nusselt and Sherwood number; but with the increase of Schmidt number $S_c$ Sherwood number is seen to increase as shown in Table-3. The magnitude of skin-friction co-efficient also decreases initially with the increase of Prandtl number

$P_r$ and then finally increase for higher value of Prandtl number $P_r > 2.5$. The effect of Prandtl number $P_r$ is seen to increase the Nuseelt number in Tabe-2.The magnitude of skin friction are seen to increase with the increase of thermal buoyancy parameter $G_r$, concentration buoyancy parameter $G_m$ and time $t$ as shown in the Table-4, Table-5 and Table-6 respectively and no effects are observed in Nusselt number and Sherwood number except time $t$. It is clearly observed from Table-6 that Skin friction coefficient, Nusselt number and Sherwood number increase with increase of time $t$.

Numerical evaluations of the results obtained by using Laplace transform technique for the typical profiles of the dimensionless velocity, temperature and concentration of the conducting fluid reported in the preceding section were performed for different existing parameters across the boundary layer. A representative set of graphical results are depicted in Fig.1-10. Fig.1, 2 & 3 demonstrate the typical profiles of the dimensionless velocity for different the magnetic parameter $M_1$, different thermal buoyancy parameter $G_r$ and different concentration buoyancy parameter $G_m$ respectively. The effects of magnetic parameter $M_1$ on the velocity distribution is shown in Fig.1 where velocity of the conducting fluid is seen to decrease with increasing the value of$M_1$. This is due to the presence of Lorentz force which retards the motion of the conducting fluid across the boundary layer. Physically, the higher of magnetic parameter $M_1$ implies the higher of the ponder motive force called Lorentz force which reduces the boundary layer thickness of the conducting fluid. So the velocity decreases with increase in $M_1$. It can be clear from Fig.2 & 3 that the effects of both thermal buoyancy parameter $G_r$ and concentration buoyancy parameter m G on the dimensionless velocity of the conducting fluid are seen to increase across the boundary layer. This is due to the physical point of view as buoyancy force increases in the upward direction and as a result induces more flow along the vertical plate/sheet which causes the velocity of the fluid increase. i.e. assisting the flow in upward direction. This Phenomenon in the flow velocity occurs at the expense of both the temperature and concentration of the conducting fluid across the boundary. Fig.4 represents the temperature profiles for various Prandtl number $P_r$ and it is observed that the effect of Prandtl number $P_r$ is to decrease the temperature of the conducting fluid across the boundary layer. The increase of Prandtl number $P_r$ leads to fall temperature in the temperature of the conducting fluid. The reason is that lower $P_r$ value has more uniform temperature distribution across the boundary layer as compared to higher $P_r$ value. This Phenomenon occurs only when the lesser values of $P_r$ are equivalent to increasing thermal conductivity. Therefore, heat is capable to diffuse away from the heated surface more quickly compare to bigger values of Prandtl number $P_r$. Fig.5 depicts the concentration profiles of the conducting fluid for the different Schmidt number $S_c$. It is obvious from Fig.5 that the effect of increasing values of $S_c$, results in an decreasing the concentration distribution. That is, concentration boundary layer thickness decreases with increasing the value of Schmidt number $S_c$. The effect of Prandtl number $P_r$ on the dimensionless profiles is shown in Fig.6. It can be seen that the velocity distribution of the conducting fluid decrease with the increase of $P_r$. Fig.7 represents the velocity distribution of the conducting fluid across the boundary layer for different Schmidt number $S_c$. It is seen that the effect of $S_c$ is to decrease the velocity of the conducting fluid. Fig.8, 9 & 10 show the velocity, temperature and concentration for different values of time $t$ respectively. It is clearly shown that with increase of time $t$; the velocity, temperature and concentration distribution are also observed to increase across the boundary layer.

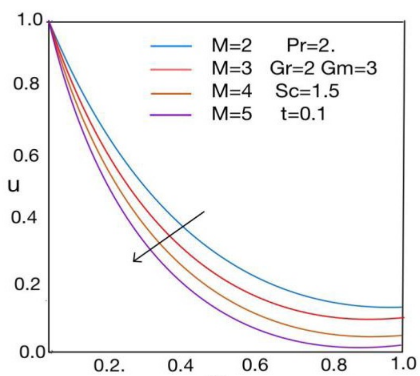


Fig.1 Velocity profiles for different magnetic parameter M

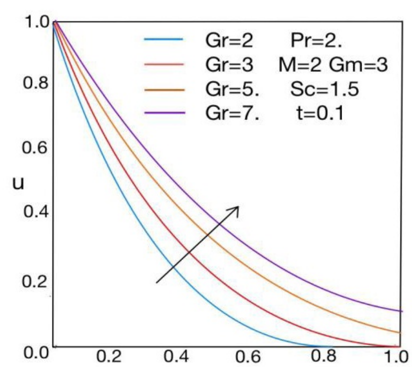


Fig.2 Velocity profiles for different Grashof number Gr

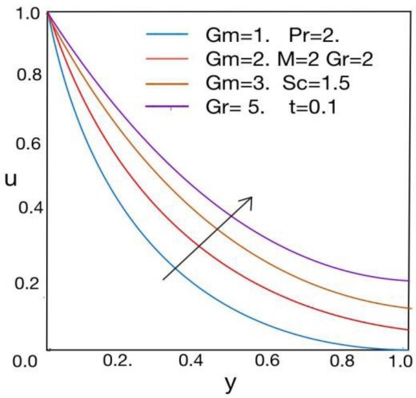


Fig.3 Velocity profiles for different concentration buoyancy parameter Gm

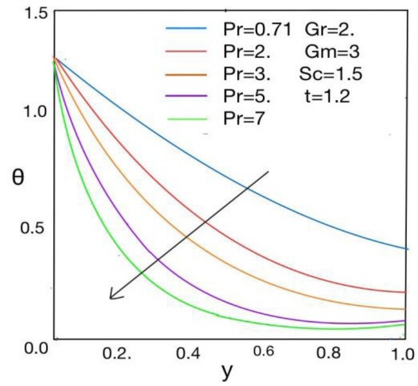


Fig.4 Temperature profiles for different Prandtl number Pr

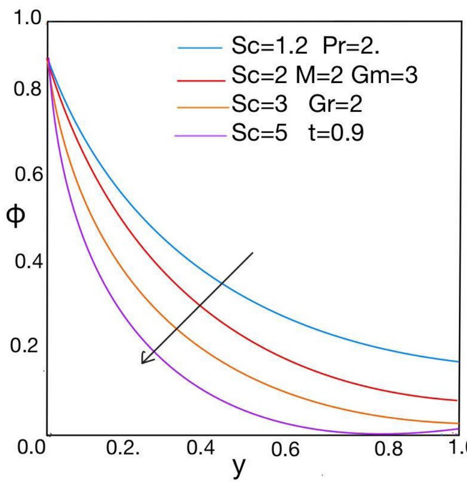


Fig.5 Concentration profiles for different Schmidt number Sc

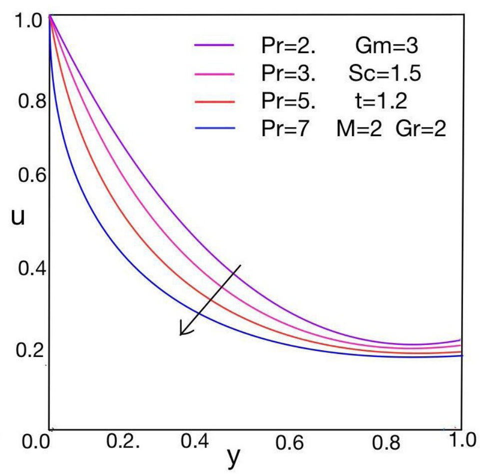


Fig.6 Velocity profiles for different Prandtl number Pr

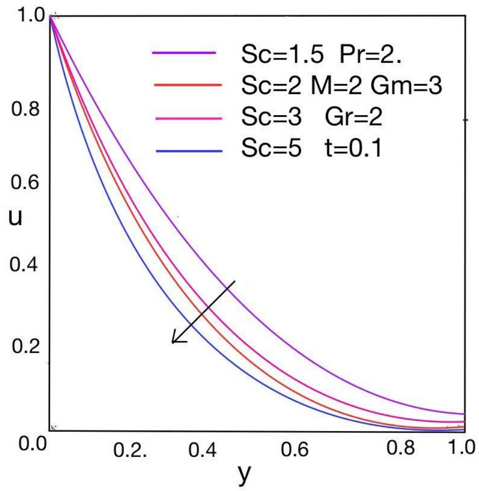


Fig.7 Velocity profiles for different Schmidt number Sc

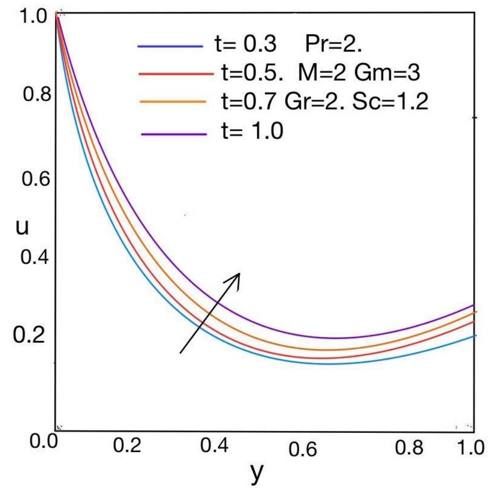


Fig.8 Velocity profiles for different time t.

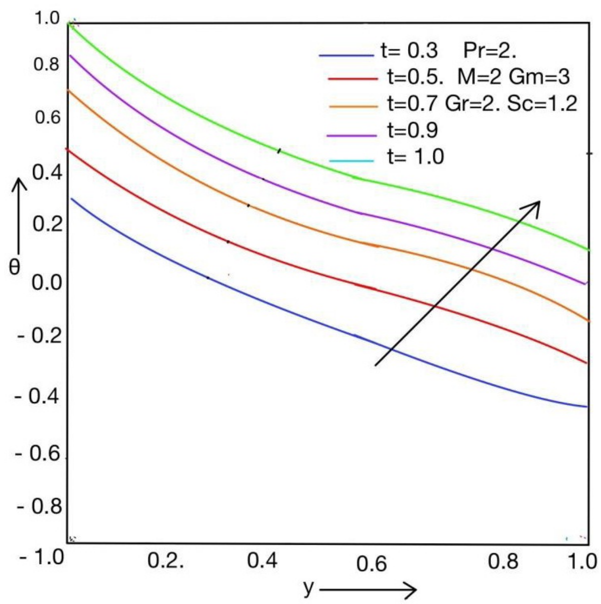


Fig.9 Temperature profiles for different time t.

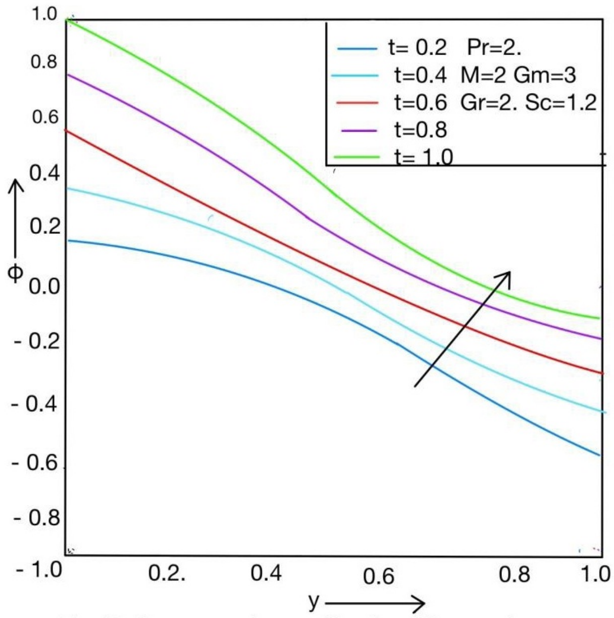


Fig.10 Concentration profiles for different time t.

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